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BSc3MatC0301 **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination
July-2025 (OLD COURSE)
MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) Answer any one out of two. (07)

1) in usual notation prove that for a>0, $\sqrt[n]{a} \to 1$ if $n \to \infty$
2) prove that if sequence $\{S_n\}$ is monotonically increasing and it is bounded Above *then $\{S_n\}$ is convergent.*

Que-1(B) Answer any one out of two. (04)

1) prove that every convergent sequence has a unique limit.
2) Discuss convergence of sequence $\{\sin\frac{n\pi}{2}\}$.

Que-1(C) Answer any one out of two. (03)

1) Discuss the convergence of sequence:
$$S_1 = \sqrt{2}, S_{n+1} = \sqrt{2+S_n} \ , n \in N$$
2) Discuss the convergence of sequence $\{S_n\}$ where
$$S_n = 1+\frac{1}{2}+\frac{1}{3}+.........+\frac{1}{n}$$

Que-2(A) Answer any one out of two. (07)

1) Define: series, geometric series.
Test the convergence of series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}.........$
2) Discuss comparision test.
Test the convergence of series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+.........$

Que-2(B) Answer any one out of two. (04)

1) Explain D'Alembert's Ratio test.
Test the convergence of series $\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+.........$
2) Explain Rabbe's test.
Test the convergence of series $1+\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+.........$

Que-2(C) Answer any one out of two. (03)

1) Find the radius of convergence for the series $\sum_{n=0}^{\infty}\frac{x^n}{n+2}$
2) Show that the series $\sum(-1)^n\frac{n}{n^3+1}$ is absolutely convergent.

Que-3(A) **Answer any one out of two.** **(07)**

1) (i) in usual notation prove that $div(\phi\bar{f}) = \phi div\bar{f} + \bar{f}.grad\phi$.

(ii) For a scalar function ϕ on D, Prove that curl(grad ϕ)= $\bar{0}$.

2) Prove that $\nabla^2(\phi\Psi) = \phi\nabla^2\Psi + 2\nabla\phi\Psi + \Psi\nabla^2\phi$

Que-3(B) **Answer any one out of two.** **(04)**

1) Prove that $\frac{1}{r}$ satisfies the Laplacian equation. Also prove that if r = |r|

Then Curl(gradr^n)= $\bar{0}$. Where $\bar{r}$ = (x,y,z).

2) in usual notation prove that div($r^n\bar{r}$) = (n+3) r^n

Que-3(C) **Answer any one out of two.** **(03)**

1) if $\bar{f}$ = (x^3,y^3,z^3) then prove that curl $\bar{f}$ = $\bar{0}$ and grad(div $\bar{f}$)= 6$\bar{r}$.

2) Define: Gradient, Laplacian operator.

Que-4(A) **Answer any one out of two.** **(07)**

1) Find the area of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ by double integral.

2) Evaluate: $\iiint_R \frac{dxdy}{\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}}$ where R is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Que-4(B) **Answer any one out of two.** **(04)**

1) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xydydx$ and hence Evaluate it.

2) Define triple integral. Find the volume of a sphere having radius a.

Que-4(C) **Answer any one out of two.** **(03)**

1) Change in spherical Coordinate $\iiint_R (x^2y)dxdydz$. Where R is $x^2+y^2+z^2 \le a^2$.

2) Define: Double integration and explain evaluation of double integration.

Que-5(A) **Answer any one out of two.** **(07)**

1) State and prove Green's theorem for a plane.

2) Define Beta function. Prove that $\beta(p,q) = \int_0^\infty \frac{y^{p-1}}{(1+y)^{p+q}}dy$. Where p,q > 0.

Que-5(B) **Answer any one out of two.** **(04)**

1) Evaluate: $\int_0^1 \frac{x^5}{\sqrt{1-x^4}}dx.$.

2) Verify stokes theorem for $\bar{F}$ = $x^2\bar{i}$+xy$\bar{j}$. Where S is a square whose sides

3) are x=0, y=0, x=a, y=a. where z=0.

Que-5(C) **Answer any one out of two.** **(03)**

1) Verify divergence theorem for $\bar{V} = xi + yj + zk$ and S is a sphere $x^2+y^2+z^2=1$.

2) Prove : $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$

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